10. MATHEMATICS – I

10.1 JNTU SYLLABUS

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UNIT-I

Theory of Matrices: Real matrices – Symmetric, skew – symmetric, orthogonal. Complex matrices: Hermitian, Skew-Hermitian and Unitary Matrices. Idempotent matrix, Elementary row and column transformations-Elementary matrix, Finding rank of a matrix by reducing to Echelon and normal forms. Finding the inverse of a non-singular square matrix using row/ column transformations (Gauss- Jordan method). Consistency of system of linear equations (homogeneous and non- homogeneous) using the rank of a matrix. Solving m x n *and* n x n *l*inear system of equations by Gauss elimination.Cayley-Hamilton Theorem (without proof) – Verification. Finding inverse of a matrix and powers of a matrix byCayley-Hamilton theorem, Linear dependence and Independence of Vectors. Linear Transformation – Orthogonal Transformation. Eigen values and eigen vectors of a matrix. Properties of eigen values and eigen vectors of real and complex matrices. Finding linearly independent eigen vectors of a matrix when the eigen values of the matrix arerepeated.Diagonalization of matrix – Quadratic forms up to three variables. Rank – Positive definite, negative definite, semi definite, index, signature of quadratic forms. Reduction of a quadratic form to canonical form.

UNIT – II

Differential calculus methods. Rolle's Mean value Theorem – Lagrange's Mean Value Theorem – Cauchy's mean value Theorem – (all theorems without proof but with geometrical interpretations), verification of the Theorems and testing the applicability of these theorem to the given function. Functions of several variables: Functional dependence- Jacobian- Maxima and Minima of functions of two variables without constraints and with constraints-Method of Lagrange multipliers.

UNIT – III

Improper integration, Multiple integration & applications: Gamma and Beta Functions –Relation between them, their properties – evaluation of improper integrals using Gamma / Beta functions Multiple integrals – double and triple integrals – change of order of integration- change of variables (polar, cylindrical and spherical) Finding the area of a region using double integration and volume of a region using triple integration.

UNIT – IV

Differential equations and applications Overview of differential equations- exact, linear and Bernoulli (NOT TO BE EXAMINED). Applications of first order differential equations – Newton's Law of cooling, Law of natural growth and decay, orthogonal trajectories. Linear differential equations of second and higher order with constant coefficients, Non-homogeneous term of the type $f(X) = e^{ax}$, Sin ax, Cos ax, and x^n, $e^{ax}V(x)$, $x^nV(x)$, method of variation of parameters. Applications to bending of beams, Electrical circuits and simple harmonic motion.

UNIT – V

Laplace transform and its applications to Ordinary differential equations Definition of Integral transform, Domain of the function and Kernel for the Laplace transforms. Existence of Laplace transform. Laplace transform of standard functions, first shifting Theorem, Laplace transform of functions when they are multiplied or divided by "t".Laplace transforms of derivatives and integrals of functions. – Unit step function – second shifting theorem – Dirac's delta function, Periodic function – Inverse Laplace transform by Partial fractions(Heaviside method) Inverse Laplace transforms of functions when they are multiplied or divided by "s", Inverse Laplace Transforms of derivatives and integrals of functions, Convolution theorem –- Solving ordinary differential equations by Laplace transforms.

TEXT BOOKS:

1. Advanced engineering Mathematics by Kreyszig, John Wiley & Sons Publishers.
2. Higher Engineering Mathematics by B.S. Grewal, Khanna Publishers.

REFERENCES:

1. Advanced Engineering Mathematics by R.K. Jain & S.R.K. Iyengar, 3rd edition, Narosa Publishing House, Delhi.
2. Engineering Mathematics – I by T.K. V. Iyengar, B. Krishna Gandhi & Others, S. Chand.
3. Engineering Mathematics – I by D. S. Chandrasekhar, Prison Books Pvt. Ltd.
4. Engineering Mathematics – I by G. Shanker Rao & Others I.K. International Publications.

5. Advanced Engineering Mathematics with MATLAB, Dean G. Duffy, 3rd Edi, CRC Press Taylor & Francis Group.
6. Mathematics for Engineers and Scientists, Alan Jeffrey, 6ht Edi, 2013, Chapman & Hall/ CRC
7. Advanced Engineering Mathematics, Michael Greenberg, Second Edition. Pearson Education.

10.2 Unit wise Planner for Academic Year 2013- 2014

Subject: Mathematics –I

Unit No.	Date Planned to complete	Date Conducted	Remarks
I			
II			
III			
IV			
V			

10.3 Session Planner

Sno	Unit	Class	Topic	Text/Reference book	Date conducted
1	I	L1	Introduction to matrices	Rf1	
2		L2	Types of matrices Real matrices – Symmetric, skew – symmetric, orthogonal.	Rf1	
3		L3	Complex matrices: Hermitian, Skew-Hermitian and Unitary Matrices. Idempotent matrix,	Rf1	
4		L4	Elementary row transformations	Rf1	
5		L5	Define rank of matrix	Rf1	
6		L6	Rank of matrix by echelon form	Rf1	
7		L7	Rank of matrix by normal form	Rf1	
8		L8	Rank of matrix by normal form using PAQ form		
9		L9	Finding Inverse of matrix using1 Gauss Jordan Method		
10		L10	Solutions of system of Linear equations	Rf1	
11		L11	Consistency of system of linear equations (homogeneous) using the rank of amatrix.	Rf1	
12		L12	Consistency of system of linear equations (non-homogeneous) using the rank of a matrix.	Rf1	
13		L13	Consistency of system of linear equations (non-homogeneous) using the rank of a matrix.	Rf1	
14		L14	Problems	Rf1	
15		L15	Solving *m* x *n and n* x *n l*inear system of equations by Gauss	Rf1	

			elimination		
16		L16	Solving *m* x *n and n* x *n l*inear system of equations by Gauss elimination	Rf1	
17		L17	Cayley – Hamilton theorem	Rf1	
18		L18	Inverse by Cayley – Hamilton theorem	Rf1	
19		L19	powers of a matrix by Cayley-Hamilton theorem	Rf1	
20		L20	Linear dependence and Independence of Vectors. Linear Transformation – Orthogonal Transformation		
21		L21	Characteristic equations , eigen value and eigen vectors	Rf1	
22		L22	Method of finding eigen values & vectors	Rf1	
23		L23	Problems	Rf1	
24		L24	Properties of eigen values and vectors of Real Matrices	Rf1	
25		L25	Properties of eigen values and vectors of Real Matrices	Rf1	
26		L26	Properties of eigen values and vectors of Complex Matrices1		
27		L27	Properties of eigen values and vectors of Complex Matrices	Rf1	
28		L28	Finding linearly independent eigen vectors of a matrix when the eigen values of the matrix are repeated.	Rf1	
29		L29	Diagonolization of a matrix	Rf1	
30		L30	Quadratic forms with problems	Rf1	
31		L31	Canonical or Normal form of Quadratic form	Rf1	
32		L32	Reduction of Quadratic to canonical form	Rf1	
33		L33	Define rank- positive ,negative, semi definite, index and signature	Rf1	
34		L34	Problems on above topics	Rf1	
35		L35	Old Question papers - solutions		
36	II	L36	Rolle's mean Value theorem	Rf1	
37		L37	Lagrange's mean value theorem.	Rf1	
38		L38	Problems on Lagrange's mean value theorem	Rf1	
39		L39	Cauchy's Mean Value theorem.	Rf1	
40		L40	Taylor's Theorem.	Rf1	
41		L41	Maclaurin's Theorem.	Rf1	
42		L42	Functions of several variables.Jacobian	Rf1	
43		L43	Functiona1l Dependence.	Rf1	
44		L44	Problems on above topic.		
45		L45	Maxima and Minima of functions of two variables with constraints.	Rf1	
46		L46	Problems on above topic.	Rf1	
47		L47	Maxima and Minima of functions of two variables without constraints.	Rf1	
48		L48	Maxima and Minima of functions of two variables without constraints(Lagranges)	Rf1	
49		L49	Old questions	Rf1	
50		L50	Objective questions	Rf1	

51	III	L51	Gamma Functions,their properties	Rf1	
52		L52	Evaluation of improper integrals using Gamma		
53		L53	Beta Functions ,their properties	Rf1	
54		L54	Gamma and Beta Functions –Relation between them,	Rf1	
55		L55	Evaluation of improper integrals using Beta functions	Rf1	
56		L56	Multiple integrals-Double integrals.	Rf1	
57		L57	Triple integrals.	Rf1	
58		L58	Change of order of integration	Rf1	
59		L59	Problems on above topic.		
60		L60	Volume of solid of revolution about co-ordinate axis.	Rf1	
61		L61	Volume about any axis, surface area of solid revolution about co-ordinate axis.	Rf1	
62		L62	Surface area of solid revolution for polarform.	Rf1	
63		L63	Surface are1a of solid revolution for parametric equation.	Rf1	
64		L64	Old questions	Rf1	
65		L65	Objective questions	Rf1	
66	IV	L66	Introduction, order and degree formation of differential equation.	Rf1	
67		L67	Introduction, order and degree formation of differential equation.	Rf1	
68		L68	Differential equations of first order and first degree. Variable separable.	Rf1	
69		L69	Homogeneous equations.	Rf1	
70		L70	Non – Homogeneous equations.	Rf1	
71		L71	Linear equations.	Rf1	
72		L72	Bernoulli's equations.	Rf1	
73		L73	Applications of Newton's Law of cooling.	Rf1	
74		L74	Law of Natural growth and decay.	Rf1	
75		L75	problems	Rf1	
76		L76	Exact differential equation.	Rf1	
77		L77	DE reducible to exact.	Rf1	
78		L78	Problems on exact and reducible to exact	Rf1	
79		L79	Orthogonal t1rajectories – Cartesian co-ordinates.	Rf1	
80		L80	Orthogonal trajectories – Polar co-Ordinate.	Rf1	
81		L81	Introduction of Non homogeneous Linear Differential equation with variable and constant coefficient.	Rf1	
82		L82	Rules for finding complimentary function.	Rf1	
83		L83	Rules for finding particular integrals R.H.S terms of type e^{ax}	Rf1	
84	V	L84	Introduction: Properties and Existence of the Laplace transform.	Rf1	
85		L85	Laplace transform of standard functions.	Rf1	
86		L86	Second shifting theorem, change of scale property.	Rf1	
87		L87	Laplace transform of periodic functions.	Rf1	

88		L88	Laplace transform of derivatives and integrals.	Rf1	
89		L89	Multiplication by t^n, Division by t	Rf1	
90		L90	Unit step function, Dirac's delta ftns.	Rf1	
91		L91	Inverse Laplace transform linear property method of partial fractions.	Rf1	
92		L92	Inverse Laplace transform derivations & integrals.	Rf1	
93		L93	Division by power's of s.	Rf1	
94		L94	Convolution theorem.	Rf1	
95		L95	Application to differential equations.	Rf1	
96		L96	Old questions	Rf1	
97		L97	Objective que1stions	Rf1	

10.4. QUESTION BANK

UNIT-I THEORY OF MATRICES
11.4.1.1 DESCRIPTIVE QUESTIONS

1. Solve the equation 3x+4y=18, 2x-y+8z+13 and 5x-2y+7=20 by matrix inversion method
2. Solve the system of equations by matrix method :$x_1 + x_2 + x_3 = 2$; $4x_1 - x_2 + 2x_3 = 6$; $3x_1 + x_2 + x_3 = -18$
3. Reduce the matrix A to normal form and hence find its rank A= $\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$
4. Determine the non-singular matrices P and Q such that PAQ is in the normal form for A. Hence find the rank of A= $\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -12 \\ 1 & -4 & 11 & -19 \end{bmatrix}$
5. Solve the equation λx +2y-2z=1; 4x +2λy-z=2; 6x +6y+z=3 for all values of λ.
6. Show that the system has no solution. (Examine consistency of the equations) $-2x + y + 3z = 12$ $x + 2y + 5z = 4$ $6x - 3y = 9z = 24$
7. Solve the matrix by using matrix inversion method $3x + 2y + 4z = 7$ $2x + y + z = 7$ $x + 3y + 5z = 2$
8. Show that the following equations are consistent and solve the $x_1 + 2x_2 - x_3 = 3$ $x_1 - x_2 + 2x_3 = 1$ $2x_1 - 2x_2 + 3x_3 = 2$ $x_1 - x_2 + x_3 = -1$
9. Using Gauss elimination with pivoting to solve the system $2x_1 + x_2 - x_3 = -1$ $x_1 - 2x_2 + 3x_3 = 9$ $3x_1 - x_2 + 5x_3 = 14$
10. Find the inverse of the matrix A= $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$

    Using Gauss elimination method.
11. Decompose the matrixA= $\begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ In to form LU and hence solve the system.
12. Reduce the following matrix to the tridiagonal form by House holder's method. (a) $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

    Find the eigen values and eigen ve1ctors of the matrix A and its inverse , where A= $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$
13. Find the eigen values and eigen vectors of the matrx $\begin{bmatrix} 6 & -2 & 0 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
14. Diagonalize the matrix $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 2 & 1 \\ -2 & -2 & -1 \end{bmatrix}$
15. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ -1 & 2 & 2 \end{bmatrix}$
16. Show thar the matrix A= $\begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ Satisfies the it's characteristic equation, Hence find A^{-1}

17 Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 0 \end{bmatrix}$ using Cayley- Hamilton theorem

18. Reduce the quadratic form to its normal from by an orthogonal reduction
$3x^2+2y^2+3z^2-2xy-2yz$

19. Reduce the quadratic form to canonical form by linear transform $x^2+y^2+2z^2-2xy+xz$

20. Reduce the quadratic form to canonical form by orthogonal transform
$3x^2+3y^2+3z^2+2xy+2xz-2yz$

21. Indicate whether the matrix is Hermitian, Skew-Hermatian or Unitary and find their eigen values and eigen vectors.

22. If A= $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 2 & 1 \\ -2 & -2 & -1 \end{bmatrix}$ find A^8

11.4.1.2 OBJECTIVE QUESTIONS

1. Which of the following matrix is scalar matrix.
(a) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (d) none

2. Which of the following matrix is upper triangular matrix
(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 0 \\ 4 & 2 & 0 \end{bmatrix}$ (d) none

3. A square matrix ‘A’ is said to be idempotent if____________
(a) $A^2 = A$ (b) $A^2 = I$
(c) $A^2 = 0$ (d) none

4. A square matrix ‘A’ is said to be involutory matrix if____________
(a) $A^2 = A$ (b) $A^2 = I$
(c) $A^2 = 0$ (d) none

5. A square matrix ‘A’ is said to be nilpotent matrix if there exist a positive integer ‘m’ such that
(a) $A^m = 0$ (b) $A^m > 0$ (c) $A^m < 0$ d) none

6. Two matrices A and B are said to be equivalent if
(a) they are of the same size (b) they are of the same rank
c) (a) and (b) (d) none

7. What is the index of the matrix A = $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

(a) 1 (b) 0 (c) 2 (d) none

8. $ABC)^{-1}$ = ____________
(a) $A^{-1}B^{-1}C^{-1}$ (b) $A^{-1}C^{-1}B^{-1}$ (c) $B^{-1}C^{-1}A^{-1}$ (d) none

9. Find the value of ‘x’ such that A is singular where A = $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -(1+x) \end{bmatrix}$

(a) 3 (b) 2 (c) 4 (d) none

10. What is the rank of the matri1x A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

a) 1 (b) 2 (c) 3 (d) none

11. What is the rank of the matrix A = $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

(a) 0 (b) 1 (c) 2 (d) 3

12. What is the rank of the matrix A = $\begin{bmatrix} 1 & a & b & 0 \\ 1 & c & d & 1 \\ 1 & a & b & 0 \\ 1 & c & d & 1 \end{bmatrix}$
    (a) 0 (b) 1 (c) 2 (d) 3

13. The rank of a matrix in Echelon form is equal to
    (a) number of non-zero rows only (b) number of non zero columns only
    (c) (a) and (b) d) none

14. Trace of a matrix A = $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 3 \\ 0 & 1 & -3 \end{bmatrix}$
    (a) 0 (b) 1
    (c) 2 (d) -3

15. The rank of a matrix in normal form is equal to the order of_____________=
    (a) unit matrix (b) symmetric matrix
    (c) Orthogonal matrix (d) none

16. Condition for inverse of a matrix A are
    (a) A should be square matrix and $|A| \neq 0$ (b) A should be square matrix
    c) $|A| \neq 0$ (d) none1

17. What is the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$
    (a) 0 (b) 3 (c) 2 (d) 1

18. If the rank of a $\begin{bmatrix} \mu & -1 & 0 \\ 0 & \mu & -1 \\ -1 & 0 & \mu \end{bmatrix}$ is 2 then μ = ________________
    (a) 0 (b) 1 (c) 2 (d) 3

19. The vectors (1, 2, 3), (3, -2, 1), (1, -6, -5) forms
    (a) linearly dependent (b) linearly independent
    (c) cannot be determined (d) none

20. The number of linearly independent solution of AX = 0 is ____________if $\rho(A) = r$ when $r \leq n$, 'n' is the number of unknowns
    (a) n – (r – 1) (b) n – (r + 1)
    (c) n – r (d) none

21. If AX = 0 has a non-zero solution if A is ____________
    (a) Singular (b) non-singular
    (c) cannot say (d) none

22 If rank of a coefficient is equal to the number of unknowns the system of equations of the form AX = 0 Possesses
    (a) Zero solution (b) finite number of solution
    (c) Infinite number of solution (d) none

23. If r < n where 'r' is the rank of the coefficient matrix 'n' is the number of unknowns then system of equations of the form AX1 = 0 posses
    (a) a unique solution (b) finite number of solution
    (c) Infinite number of solution (d) none

24. The solution of system of equations
    $x + 3y + 2z = 0, x + 4y + 3z = 0, x + 5y + 4z = 0$
    (a) x = y = z =1 (b) x = z = 1, y = -1
    (c) x =0, y = 2, z = -3 (d) x =0, y = 2, z = 4

25. The necessary and sufficient condition that the system of equation AX = B is consistent

(a) ρ (AB) = ρ (A)   (b) ρ (AB) < ρ (A)
(c) ρ (AB) > ρ (A)   (d) none

26. The equation AX = B has a unique solution if A is _________________
a) singular   (b) non-singular
(c) cannot say   (d) none

27. The system of equation AX = B has no solution if
a) ρ (A) = ρ (AB)   (b) ρ (A) $\neq$ ρ (AB)
(c) cannot say   (d) none

28. The system of equation AX = B has a unique solution if
a) ρ (A) = ρ (AB) = r and r = n   (b) ρ (A) = ρ (AB) = r and r < n
(c) ρ (A) = ρ (AB) and r = n   (d) ρ (A) = ρ (AB) and r < n

29. The system of equation AX = B have infinite number of solution if
a) ρ (A) = ρ (AB) = r and r < n   (b) ρ (A) = ρ (AB) = r and r = n
c) ρ (A) = ρ (AB) = r and r > n   (d) none

30. For what values of a and b the equations
$x+2y+3z=4, x-3y+4z=5, x+3y+az=b$ have no solution
(a) a = 6, b $\neq$ 5   (b) a = 5, b = 5   c) a = 3, b = 5   (d) a = 4, b $\neq$ 5

31. For what values of a and b the equations
$x+2y+3z=4, x-3y+4z=5, x+3y+az=b$ has a unique solution
(a) a = 4, b = 5   (b) a $\neq$ 4, b =5   (c) a = 5, b = 4   (d) a $\neq$ 4, b =4

32. For what values of a and b the $x+2y+3z=4, x-3y+4z=5, x+3y+az=b$
have an Infinite number of solutions
(a) a = 4, b = 5   (b) a 1$\neq$ 4, b$\neq$ 5   (c) a $\neq$ 4, b = 5   (d) a = 4, b $\neq$ 5

33. If rank (A) = 2, rank(B) = 3, then rank (AB) = 6. this statement is
(a) True   (b) false   (c) cannot say   (d) none

34. A square matrix A is invertible if
(a) A is singular   (b) A is non-singular   (c) cannot say   (d) none

35. If there are 4 equations in 5 unknowns and the rank of the coefficient matrix is 2, the then the system AX = B will have
(a) a unique solution   (b) two solutions   (c) Infinite number of solution   (d) none

36. To solve the system AX = B by LU decomposition method, we have to solve two system of equations_____________
(a) LY = B, LX = B   (b) LY = U, UX = Y   (c) LY = B, UX = Y   (d) LY = B, UX = Y

37. T he solution o f the e equations x - y - z =0 , x + y +2 z =4 and y + z =1 i s
(a) 2 , 1, 3   (b) 1 , - 1 , 2   (c) 1 , 1, 1   (d) 1 , 2, 0

38. The system of equation s x + y + z = 1, x + 2 y = 3, y – z = 2 w i l l h ave
(a) No solutions   (b) Infinite no of solutions   (c) Unique solution   (d) two solutions

39. If the system of equations x + 2y + 3z =0 , 3 x+ 4 y+ 1 0z = 0, 2x + 4y + az =0 , possesses a non trivial solution then the value of a is
(a)6   (b) 1   (c) 3   (d) 2

40. The eigen values of a real symmetric matrix are
(a) imaginary   (b) 0   (c) real   (d) can't be decided

41. The diagonal elements of a skew symmetric matrix are
(a) real   (b) imaginary   (c) 1   (d) 0

42. If two eigen vectors of a symmetric matrix are $\begin{bmatrix}1\\-1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\0\\-1\end{bmatrix}$ then the third eigen vector is

(a) $\begin{bmatrix}1\\-1\\1\end{bmatrix}$ 1(b) $\begin{bmatrix}-1\\1\\1\end{bmatrix}$ (c) $\begin{bmatrix}1\\2\\1\end{bmatrix}$ (d) $\begin{bmatrix}-1\\-2\\2\end{bmatrix}$

43. If a square matrix A is orthogonal then A^{-1} is
(a) symmetric   (b) skew-symmetric   (c) orthogonal   (d) unitary

44. If A = $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is an orthogonal matrix then A^{-1}

(a) $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$ (c) $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & 2 & 1 \end{bmatrix}$

45. The matrix $\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$ is

(a) Symmetric (b) diagonal (c) orthogonal (d) unitary

46. The determinant of an orthogonal matrix is

a) 0 (b) > 1 (c) < 1 (d) ± 1

47. A square matrix A is unitary if

(a) $A^T = A^{-1}$ (b) $(\overline{A})^T = A^{-1}$ (c) $\overline{A} = A^{-1}$ (d) $-A^T = A^{-1}$

48. The matrix $\begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/2 & -1/\sqrt{2} \end{bmatrix}$ is

(a) symmetric (b) diagonal (c) orthogonal (d) unitary

49. The eigen values of a Hermitian matrix are

(a) real (b) 0 (c) imaginary (d) 1

50. The matrix $\begin{bmatrix} 1 & 1-2i & 3+i \\ 1+2i & 2 & 3-2i \\ 3-i & 3+i & -1 \end{bmatrix}$ is

(a) symmetric (b) skew-symmetric (c) hermitian (d) skew-hermitian

51. If A is a Hermitian matrix then iA is

a) orthogonal (b) unitary (c) hermitian (d) skew-hermitian

52. The eigen values of $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are

(a) i, -i (b) 1, -1 (c) 1, 1 (d) -1, -1

53. The symmetric matrix associated with the quadratic form $x^2 + 3y^2 - 8xy$ is

(a) $\begin{bmatrix} 1 & -4 \\ -4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -8 \\ -8 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 4 \\ 4 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

54. The symmetric matrix associated with the quadratic form $x^2 - y^2 + 2z^2 + 2xy - 4yz + 6xz$ is

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -2 \\ 1 & -2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & -2 \\ 3 & -2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

55. The quadratic form associated with the symmetric matrix $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -2 \\ 2 & -2 & 3 \end{bmatrix}$ is

(a) $x^2 + 2y^2 + 3z^2 - 2xy + 4xz - 4yz$ (b) $x^2 - 2y^2 + 3z^2 + 2xy + 4xz + 4yz$
(c) $x^2 + 2y^2 + 3z^2 - 2xy - 4xz - 4yz$ (d) $x^2 + 2y^2 + 3z^2 - 2xy - 4xz + 4yz$

56. If the eigen values of A are -1, 2, 3 then the index and signature of X^TAX are

(a) 1, 1 (b) 1, 2 (c) 2, 1 (d) 2, -1

57. If the eigen values of A are 1, 1, 4 then the nature of the quadratic form X^TAX are

(a) indefinite b) positive definite (c) negative definite (d) positive semi definite

58. If the eigen values of A are -3, 3, 5 then the nature of the quadratic form X^TAX are

(a) positive semi definite (b) positive definite (c) negative definite (d) indefinite

59. The nature of the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$

(a) positive definite b) positive semi definite (c) negative definite d) negative semi definite

60 If A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then the nature of the quadrature form X^TAX is

(a) positive definite (b) positive semi definite (c) negative definite (d) indefinite

61. If A = $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ then the index and signature of X^TAX

(a) 3, 3 (b) 3, 2 (c) 3, 1 (d) 2, 1

62. The index and signature of $2x^2 + 2y^2 + 2z^2 + 2yz$ are

(a) 3, 3 (b) 3, 2 (c) 3, 1 (d) 2, 1

63. The index and signature of $x^2 + 3y^2 + 3z^2 - 2yz$ are

(a) 3, 3 (b) 3, 2 (c) 3, 1 (d) 2, 1

64. If A is asymmetric singular matrix and two of eigen values arer positive then the nature of X^TAX is

(a) positive definite b) positive semi definite (c) negative definite d) indefinite

65. The diagonal elements of a Hermitian matrix are ……….

(a) real (b) imaginary (c) all zero (d) none

66. The necessary and sufficient condition for a square matrix to be hermitian is that………

(a)$A^{\theta} = A$ (b) $A^{\theta} = -A$ (c) $A^{\theta} = A^2$ (d)none

67. The diagonal elements of a skew-hemitian matrix are ………

(a) real (b) imaginary (c) either imaginary or zero (d)none

68. If A is a hermitian matrix ,then iA is …………..

(a)Hermitian (b) skew-Hermitian (c)skew-symmetric (d)none

69. The matrix A = ? ???? ??? ? / −??? ? ???? ? is

(a) Unitary (b)Orthogonal (c)Involutary (d)none

70. Inverse and transpose of a Unitary matrix is

(a)Unitary (b)Hermitian (c)Orthogonal (d)none

71. the Eigen values of real symmetric matrix are

(a) Imaginary (b)real (c) all zero (d)Either purely imaginary or zero

72. Eigen values of Skew-Hermitian matrx are

(a) Imaginary (b)real (c) all zero (d)Either purely imaginary or zero

73. The Eigen values of Orthogonal matrix are of

(a) $|\lambda| <> 1$ (b) $|\lambda| = 1$ (c) $\lambda^2 = 1$ (d)none

74. Eigen values of the matrix ?$\begin{matrix} 3-4i & 2 \\ 2 & 3+4i \end{matrix}$? are

(a) -3,7 (b) 3,7 (c) 3i,7i (d)2,2

75. A homogeneous expression of second degree in any number of variables is said tobe a

(a) Quadratic form (b) Matrix polynomial (c) Linear transformation (d)none

76. Quadratic form corresponding to the matrix $\begin{matrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{matrix}$ is

(a) $y^2+2z^2+10xy+12yz-2zx$ b) $y^2-2z-10xy-12yz+2zx$ (c)$y^2+2z^2+10xy-12yz-2zx$

77. The matrix of the Quadratic form $2x_1^2+3x_2^2+4x_3^2+5x_4^2$ is

(a) Diag [2,-3,4,5] (b) Diag [2,3,4,5] (c) Diag[-2,-3,-4,-5](d)none

78. The number of positive terms in the canonical form of a Quadratic form is called the ……….. of the Quadratic form.

(a)Index (b) Signature (c) Rank (d) none

79. If A = $\begin{matrix} 1 & 1+i & -2i \\ 1-i & 0 & 2+3i \\ 2i & 2-3i & -2 \end{matrix}$ is a **…………………**matrix

80. If A = $\begin{matrix} 2-3i & 4 & 3i \\ 0 & 2i & 6 \end{matrix}$; then A^{θ} = **………………………**

-3i 6 6+2i

81. The linear transformation y = Ax is said to be orthogonal if **………………..**
82. A and B are Hermian matrices then AB-BA is **……………….**
83. Every hermitian matrix A can be written as A= P+iQ where P and Q are **…………………………….** Matrices.
84. The matrix A = a+ic -b+id is Unitary iff **…………………….**
    b+id a-ic
85. The Eigen values of the matrix A= 1 1+i are**………………..**
    1-i 2
86. If P is a real orthogonal matrix and D is a real diagonal matrix such that P^{-1} AP=D Then A is**……………………….**matrix.
87. If the eigen values of the matrix A are 3,6,9 and P is an orthogonal matrix then P^{-1} AP =**…………………… .**
88. Signature of the Quadratic form is**…………………….**
89. The index of the Quadratic form is**……………………..**
90. If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of the matrix of a Quadratic form , then yhe canonical Form is **……………………………..**
90. The matrix of the Quadratic form $ax^2+by^2+cz^2+2fyz+2gzx+2hxy$ is **…………………**.
91. If $2x^2+3y^2+6z^2$ is the canonical form of a Quadratic form then Rank=**……….. ,** signature=**…………..**
92. If the eigen values of the matrix A are 3,6,9 where A is matrix of the Quadratic form Then nature of the quadratic form is**………………**
93. Sylverster law is **…………………………………….**
94. The determinant of a Unitary matrix is of unit modulus (true/false)
95. P^{-1} AP =D is the Orthogonal transformation then A is Skew-symmetric matrix. (true/false)
96. the index of the Quadratic form is the number negativeterms in the canonical form (true/false80.
97. A= 1 -1 is the matrix of the Quadratic form $x^2-2xy+2y^2$ (true/false)
    -1 2
98. Inverse and Transpose of unitary matrix is unitary. (true/false)

11.4.1.3 TUTORIAL TOPICS

1. Show that A=$\begin{bmatrix}1&2&3\\2&-1&4\\3&1&1\end{bmatrix}$ satisfies the matrix equation $A^3 - A^2 - 18A - 30 = 0$.

2. Find the rank of $\begin{bmatrix}1&2&3&0\\2&4&3&2\\3&2&1&3\\4&8&7&5\end{bmatrix}$

3. For what values of λ & μ do the system of equations
   $x + y + z = 6$
   $x + 2y + 3z = 10$
   $x + 2y + \lambda z = \mu$ have
   (a) No solution (b) Unique solution (c) more than one solution

4. Prove that if the equation x=ay + z; y=-2+ax; z=x+y are consistent then $a^{3+}3=0$

5. Find non –singular matrices P & Q such that PAQ is in the normal form for the matrix A=$\begin{bmatrix}1&-1&-1\\1&1&1\\5&4&3\end{bmatrix}$

6. If A=$\begin{bmatrix}1&1&2\\1&2&3\\0&-1&3\end{bmatrix}$ Find the non singular matrices P and Q such that PAQ is normal form.

7. Discuss the solution of x +2y + 3z=ax; 3x + y + 2z=ay; 2x + 3y + z=az for all values of a.

8. the system of equations using simple elimination on process
$x + y + z = 6$ 1. $2x - y + 3z = 4$ $4x + 5y - 10z = 13$

9. Find the inverse in the following
(a) $\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 6 & 4 \\ 0 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

10. Find the inverse of the matrix

A=$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$Using Gauss elimination method.

11. Reduce the following matrix to the tridiagonal form by House holder's method.
a) $\begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$

12. Solve the system of equations
$83x + 11y - 4z = 95$ $7x + 52y + 13z = 104$

13. Find Eigen values and Eigen vectors of A=$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

14. Verify Cayley – Hamilton theorem for A=$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}$

15. Diagonalise the matrix A=$\begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$

16. Using Cayley – Hamilton theorem find the inverse of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

17. If A=$\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$ verify that A*A is a Hermitian matrix

18. Reduce $3x^2 + 3z^2 + 4xy + 8xz + 8yz$ into canonical form.

19. Reduce $3x^2 - 3y^2 - z^2 - 4xy + 8xz + 12yz$ into canonical form by orthogonal transformations and also find its rank, index and signature.

20. Find the Eigen values of A=11$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and hence reduce 2xy +2yz +2zx into canonical form.

21. Using Cayley – Hamilton theorem find the inverse of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

22. Prove that A=$\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is unitary.

23. Prove that eigen values of Hermitian matrix are real.

UNIT-II DIFFERENTIAL CALCULUS METHODS

10.4.2.1 DESCRIPTIVE QUESTIONS

1. Verify the Rolle's Theorem for the function $f(x) = (x-a)^m (x-b)^n$ in [a, b].

2. Verify the Rolle's Theorem for following the function.
   a) $f(x)=e^x(\sin x - \cos x)$ in [Π/4,5Π/4]
   b) $f(x)=x(x+3)e^{-x/2}$ in [-3,0]
3. Verify the Lagrange's mean value theorem for the following functions.
   f(x) = Log x in [1, e]. . $f(x) = x^3 - x^2 - 5x+3$ in [0,4]
4. Verify the Cauchy's mean value theorem of the flowing
   . f(x)= √x g(x) = 1/√x in [a, b] $f(x = e^x$ $g(x) = e^{-x}$ in [a, b]
5. Using mean value theorems show that…
6. $\pi/4 + 3/25 < \tan^{-1} 4/3 < \pi/4 + 1/6$
7. $\pi/3 - 1/5\sqrt{3} > \cos^{-1} 3/5 > \pi/3 - 1/8$
8. x/1+x <log(1+x) <x
9. Explain about algebraic and geometrical interpretation of Rolle's Theorem.
10. Explain about algebraic and geometrical interpretation of Lagrange's theorem.
11. Find Maclaurin's theorem with Lagrange's form of remainder for
    f(x) = cosx.
12. If $u = x^2 - 2y$; v= x+y+z; w = x-2y+3z find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$
13. If u = x+y+z, uv = y+z, uvw =z, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.
14. Expand $x^2y+3y-2$. in powers of x-y and y+2 upto the terms of 3rd degree. Using taylor's theorem.
15. Find the maximum and minimum values of $x^3 + y^3 + 3axy$.
16. Examine for minimum and maximum values of sinx+siny+sin(x+y).
17. A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material of its construction.
18. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid
    $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
19. In a plane ΔABC, find the maximum value of cos A cos B cos C.
20. Find the minimum value of $x^2 + y^2 + z^2$ when ax+by+cz = p.

10.4.2.2 OBJECTIVE QUESTIONS

1. Lagrange Mean Value Theorem is a Special case of
   a) Rolle's Theorem b) Cauchy's Mean Value Theorem
   c) Taylor's Theorem d) Taylor's series
2. The result, "If $f'(x) = 0 \forall x$ in $[a,b]$ then $f(x)$ is a constant in $[a,b]$" can be obtained from
   a) Rolle's Theorem b) Lagrange Mean Value Theorem
   c) Cauchy's Mean Value Theorem d) Taylor's Theorem
3. The first three non-zero terms in the expansion of $e^x \tan x$
   a) $x + x^2 + \frac{1}{3}x^3$ b) $x + \frac{1}{3}x^3 + \frac{2}{5}x^5$ c) $x + x^2 + \frac{5}{6}x^3$ d) $x + \frac{1}{3}x^3 + \frac{1}{6}x^5$
4. In the expansion of $\tan x$ and $\tan^{-1} x$, considering first three non-zero terms
   a)The first three non-zero terms are same b)The first two non-zero terms are same
   c) All coefficients are different d) First two coefficient are same

10.4.2.3 TUTORIAL TOPICS

1. Verify the Rolle's Theorem for the function $f(x) = (x-a)^m (x-b)^n$ in [a, b].
2. Verify the Rolle's Theorem for following the function.
   a) $f(x)=e^x(\sin x - \cos x)$ in [Π/4,5Π/4] b) $f(x)=x(x+3)e^{-x/2}$ in [-3,0]
3. Using mean value theorems show tha $\pi/4 + 3/25 < \tan^{-1} 4/3 < \pi/4 + 1/6$
4. Find Maclaurin's theorem with Lagrange's form of remainder for f(x) = cosx.
5. A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material of its construction.

6. Expand $x^2y+3y-2$. in powers of x-y and y+2 upto the terms of 3rd degree. Using taylor's theorem.
7. Find the minimum value of $x^2 + y^2 + z^2$ when ax+by+cz = p.

UNIT-III IMPROPER INTEGRATION, MULTIPLE INTEGRATION & APPLICATIONS
10.4.3.1 DESCRIPTIVE QUESTIONS

1. Find the perimeter of the loop of the curve $3ay^2=x(x-a)^2$
2. Find the perimeter of the cardioid r = a(1- cosθ)
3. Find the volume of solid obtained by revoliving one are of the cycloid x = a(θ+sinθ), y = a(1+cosθ) about x-axis.
4. Find the volume of the solid obtained by revolving the cardioid r = a (1+cosθ) about the initial line.
5. A sphere of radius 'a' units is divided into two parts by a plane distant a/2 form the centre. Show that the ration of the two volumes of the two parts is 5:27.
6. Find the surface area generated by the revolution of an arc of the catenary y = c cosh x/c about the x-axis.
7. Find the surface area of the solid generated by evolving the curve $a^2y^2 = x^2(a^2- x^2)$ about x-axis.
8. Show that the whole length of the curve $x^{2/3} +y^{2/3} =a^{2/3}$ is 6a.
9. Find the volume of the solid generated by revolving the ellipse $x^2/a^2+ y^2/ b^2=1$ about the major axis.

10 Find $\int_0^{\pi/2}\left(\sqrt{\tan\theta}+\sqrt{\sec\theta}\right)d\theta$

11 Express $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx$ as a beta function

12. Prove that $\int_0^1 x^m(\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive integer and m >-1

13. Prove that $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1}dy = \frac{\Gamma(p)}{q^p}$ where q > 0 p >0

14. Express the following integrals in terms of gamma functions:

a) $\int_0^1 \frac{1}{\sqrt{1-x^n}}$ b) $\int_0^\infty x^n e^{-a^2x^2}dx$

15. Prove that $\int_0^\infty e^{-x^{1/m}}dx$ = m Γ(m)

16. Show that $\int_0^\infty \sqrt{x}e^{-x^3}dx = \frac{\sqrt{\pi}}{3}$

17. Show that $\int_0^{\pi/2} \sin^2\theta \cos^4\theta$ = Π/32

18. Prove that $\int_0^1 \frac{xdx}{\sqrt{1-x^5}} = \frac{1}{5}\beta\left(\frac{2}{5},\frac{1}{2}\right)$

19. Express $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of gamma function and evaluate

$$\int_0^1 x^5(1-x^3)^{10}\,dx$$

20. Show that Γ(1/2) =√Π

21. Establish a relation between beta and gamma functions

22. Prove that β(m, n) = β(m+1, n) + β(m, n+1)

10.4.3.2 OBJECTIVE QUESTIONS

1. The area bounded by the curve $y^2 = x-1$ and $y = x-3$ is
   a)3 b)7/2 c)9/2 d)7/3

2. The volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is
   a) $\frac{abc}{2}$ b) $\frac{abc}{3}$ c) $\frac{abc}{6}$ d) $\frac{24}{abc}$

3. For $\int_0^\infty \int_x^\infty f(x,y)dxdy$, the change of order is
   a) $\int_x^\infty \int_0^\infty f(x,y)\,dxdy$ b) $\int_0^\infty \int_y^\infty f(x,y)\,dxdy$
   c) $\int_0^\infty \int_0^y f(x,y)\,dxdy$ d) $\int_0^\infty \int_0^x f(x,y)\,dxdy$

4. The value of the integral $\int_{-2}^{2} \frac{1}{x^2}dx$ is
   a)0 b)0.25 c)1 d)∞

5. The value of $\int_0^1\int_0^1 (x+y)dxdy$ is
   a)0 b)1 c)2 d)3

6 On changing the order of integration, the double integral $\int_0^{2a}\int_0^{x^2/4a}(x+y)^3\,dydx$ becomes

a) $\int_0^a\int_{\sqrt{4ay}}^{2a}(x+y)^3\,dxdy$ b) $\int_0^a\int_{\sqrt{2ay}}^{2a}(x+y)^3\,dxdy$ c) $\int_0^a\int_{\sqrt{4ay}}^{4a}(x+y)^3\,dxdy$ d)None

7. If the region R is bounded by x=0,y=0,x+y=1 and if the horizontal strip is considered the x limits in $\iint_R f(x,y)dxdy$ are

a)0,1 b)0,1-y c)0,1-x d)None

8. If the region R is bounded by $y=x^2$ and $y=x$ and if the vertical strip is considered the y limits in $\iint_R f(x,y)dydx$ are

a)0,1 b)0,x c)x^2 ,x d)Non

9. $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$ is equivalent to

a) $\int_0^\infty\int_0^\infty \frac{e^{-y}}{y}dxdy$ b) $\int_0^\infty\int_0^y \frac{e^{-y}}{y}dxdy$ c) $\int_x^\infty\int_0^\infty \frac{e^{-y}}{y}dxdy$ d)None

10. The volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ can be obtained by evaluating

a) $\int_0^a\int_0^{b\sqrt{1-x^2/a^2}} zdydx$ b)4 $\int_0^a\int_0^{b\sqrt{1-x^2/a^2}} zdydx$ c)8 $\int_0^a\int_0^{b\sqrt{1-x^2/a^2}} zdydx$ d)None

11. The area of the region which is outside r =1 and inside r = 1+cos θ can be obtained by evaluating

a) $\int_0^\pi\int_1^{1+\cos\theta} rdrd\theta$ b) $\int_0^{2\pi}\int_1^{1+\cos\theta} rdrd\theta$ c) $\int_0^{\pi/2}\int_1^{1+\cos\theta} 2rdrd\theta$ d)None

12 The value of ϒ(1) is

a) 0 b) 1 c) 2 d) -1

13 The value of ϒ(1/2) is

a) 0 b) 2 c) √∏ d) -√∏

14 The Reduction formula for ϒ(n) (i.e.)ϒ(n+1) is

a) nϒ(n) b) ϒ(n) c) ϒ(n+2) d) 0

15 The value of $\int_0^{\infty} e^{-x^2} dx$

a) √∏ /2 b) √∏ c) 2/√∏ d) none

16 The relation between β and ϒ functions is

a) β(m, n) = (ϒ(m)ϒ(n))/ϒ(m+n) b) β(m, n) = ϒ(m)ϒ(n)

c) β(m, n) = ϒ(m+n) d) none

17 The value of $\int_0^{\pi/2} \sin^{10}\theta d\theta$

a) ∏/256 b) 63∏/256 c) 65 ∏/256 d) none

18 $\int_0^{\pi/2} \cos^9\theta d\theta$

a) 2.4.6.8/ 1.3.5.7.9 b) 2 c) 0 d) none

19 $\int_0^{\pi/2} \sin^6\theta \cos^7\theta d\theta$

a) 1 b) 2 c) 2^4/3.7.11.13 d) 3^4/5.6.7

20 $\int_0^{2\pi} \sin^8\theta d\theta$

a) 35 ∏/64 b) 35 /64 c) ∏/64 d) none

10.4.3.3 TUTORIAL TOPICS

1. Evaluate ∭xyz dxdydz over the positive octant of the sphere $x^2+y^2+z^2=a^2$

2 Show that $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1}\beta(m+1,n+1)$

3. Show that $\int_0^{\infty} x^4 \; e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$

4. Show that $\beta(m,n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$

5 Evaluate $\int_0^1 x^3 \sqrt{1-x}\,dx$ using $\beta - \Gamma$ functions.

UNIT – IV DIFFERENTIAL EQUATIONS AND APPLICATIONS
10.4.4.1 DESRIPTIVE QUESTIONS

1. Form the differential equation of the family of circles $(x-a)^2 + (y-b)^2 = r^2$ where a and b are parameters and r is a constant. 1Eliminate the arbitrary constants and obtain the DE for following.
2. $x^2 + y^2 + 2gx + 2fy + c = 0$
3. tanx tany = c (c).
4. x tan(y/x) = c (c).
5. $y = \frac{a+x}{x^2+1}$ (a).
6. $y = e^x(c_1 \cos 2x + c_2 \sin 2x)$ (c_1, c_2).
7. Find the equation of the curve passing through the point (0, -1) and whose differential equation is $(1 + y^2)dx + (1 + x^2)dy = 0$.
8. Solve $y - x\, dy/dx = a(y^2 + dy/dx)$.
9. Solve x dy/dx +coty = 0 if y = π/4 when x = √2.
10. solve $(x^2 + 1) y^1 + y^2 + 1 = 0$, y(0) = 1.
11. Solve $(x^2 + y^2)dx = 2xy\, dy$.
12. Solve $xdy - ydx = \sqrt{(x^2 + y^2)}\, dx$ given that y = 1 when x = √3.
13. Solve dy/dx = y/(x+√xy).
14. Solve $(x^2 - 2xy + 3y^2)dx + (y^2+6xy- x^2)dy = 0$.
15. Solve (x cos (y/x) + y sin (y/x))y –(y sin (y/x) – xcos (y/x))xy^1=0.
16. Solve (3y+2x+4)dx –(4x + 6y + 5) dy = 0.
17. Solve (x+y) (dx-dy) = dx+dy.
18. Solve $x(1- x^2)dy/dx + (2x^2 - 1)y = x3$.
19. Solve $(2x - 4y + 5)y^1 + (x-2y+3) = 0$.
20. Solve xdx+y dy = $\frac{a^2(xdy - ydx)}{x^2 + y^2}$.
21. $(2x+e^y)dx + xe^y\, dy = 0$.
22. Solve x dy/dx + y = logx.
23. Solve $dy/dx - 2y/(x+1) = (x+1)^3$.
24. Solve $(1+ y^2)dx = (\tan^{-1} y - x)dy$.
25. Solve dr + (2r cotθ + sin2θ)dθ=0.
26. Solve dy/dx + y/(xlogx) = (sin2x)/logx.
27. Solve coshx dy/dx + y sinh x = $2\cosh^2 x \sinh x$.
28. Solve $xy^1+y+4 = 0$.
29. Solve $(x+1)\, dy/dx - ny = e^x (x+1)^{n+1}$.
30. Solve $x\, dy/dx + y = x^3 y^6$.
31. Solve $dy/dx + y\tan x = y^2 \sec x$.
32. Solve $dy/dx - \tan y/(1+x) = (1+x)e^x \sec y$.
33. Solve $(1- x^2)\, dy/dx + xy = y^3 \sin^{-1}x$.
34. Solve $dy/dx + y \cos x = y^3 \sin 2x$.

Find the Orthogonal trajectories of the following family of curves.

35. $y = ax^{2.}$

36. $y^2 = 4a(x+a)$.
37. $x^2 + y^2 - ax = 0$.
38. $x^2/(a^2+\lambda) + y^2/(b^2+\lambda) = 1$.
39. $r = a(1-\cos\theta)$.
40. $r^n \sin n\theta = a^n$.
41. $r = 2a/(1+\cos\theta)$.
42. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour what was the value of N after 1½ hours.
43. Uranium disintegrates at a rate proportional to the amount present at any instant. If m_1 and m_2 are grams of uranium that are present at times T_1 and T_2 respectively, find the half – life of uranium.
44. The rate at which bacteria multiply is proportional to the instantaneous number present. If the original number doubles in 2 hours, in how many hours will it be triple?
45. A body is originally at 80^oC and cools down to 60^oC in 20 minutes. If the temperature of the air is 40^oC, find the temperature of the body after 40 minutes.
46. Solve $4y^{111} + 4y^{11} + y^1 = 0$.
47. Solve $(D^4 + 18D^2 + 81)y = 0$. $D = d/dx$.
48 Solve $y^{11} + y^1 - 2y = 0$ y(0) $y(0) = 4$, $y^1(0)-1$.
49 Solve $(D^3+1)y = \cos(2x-1)$.
50 Solve $(D^3+1)y = 3+5e^x$.
51 Solve $y^{11} - 4y^1 + 3y = 4e^{3x}$, $y(0) = 1$, $y^1(0) = 3$.
52 Solve $y^{11} + 4y^1 + 4y = 4\cos x + 3\sin x$, $y(0) = 3$.
53 Solve $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x + \sin 2x$.
54 Solve $(x^4D^3 + 2x^3D^2 - x^2D + x)y = 1/x$.
55 Solve $[(1+x)^2 D^2 + (1+x) D + 1]y = \sin 2(\log(1 + x))$.
56 Solve $(x^2D^2 - xD + 2)y = x \log x$.
57 Solve $(x^2D^2 - 2xD - 4)y = x^2 + 2 \log x$.
58 Solve $(x^3D^3 + 2x^2D^2 + 2)y = 10(x+1/x)$
59 Without using variation of parameters solve $(D^2+a^2)y = \tan ax$.
60 Solve by method of variation of parameters.
61 $(D^2+4)y = \tan 2x$.
62 $(D^2+1)y = 1/e^x-1$.
63 $(D^2+n^2)y = \text{Sec } nx$.
64 $(D^2 + 3D+2)y = e^{2x}+x^2$.

10.4.4.2 OBJECTIVE TYPE QUESTIONS

1. For the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$,which of the following is not applicable

   a) It is Bernoulli equation b) It is Homogeneous c) It is not exact d) Solution is $y = 2x^2/(1+x^2)$

2. The particular solution of the equation $y' \sin x = y In y$ satisfying the initial condition $y(\pi/2) = e$ is

   a) $e^{\tan(x/2)}$ b) $e^{\cot(x/2)}$ c) $In \tan(x/2)$ d) $In \cot(x/2)$

3. The initial value problem $x\frac{dy}{dx} = y, y(0) = 0, x \geq 0$ has

   a) No solution b)A unique solution c) Exactly two solutions d) Unaccountably many solutions

4. For the differential equation $xy' - y = 0$ which of the following is not an integrating factor?

   a) $1/x^2$ b) $1/y^2$ c) $1/xy$ d) $1/(x+y)$

5. An integrating factor for $ydx - xdy = 0$ is

   a) *x/y* b) *y/x* c) $1/x^2y^2$ d) $1/(x^2+y^2)$.

6. The orthogonal trajectories of a family of parabolas $ay = x^2$ is

a)Circle b)Hyperbola c)Ellipse d)Cubical Parabola

7. The orthogonal trajectories of the system given by $r = a\theta$ is

a) $r^2 = de^{\theta}$ b) $r = de^{\theta}$ c) $r^2 = de^{-\theta^2}$ d) $r^2.e^{-\theta^2} = d$

8. The particular integral of $\frac{d^2y}{dx^2} + y = \cos x$ is

a) $\frac{1}{2}\sin x$ b) $\frac{1}{2}\cos x$ c) $\frac{1}{2}x\cos x$ d) $\frac{1}{2}x\cos x$

9. The general solution of the differential equation $(D^2+1)^2 y = 0$ is

a) $c_1\cos x + c_2\sin x$ b) $(c_1 + c_2x)\cos x + (c_3 + c_4x)\sin x$

c) $c_1\cos x + c_2\sin x + c_3\cos x + c_4\sin x$ d) $(c_1\cos x + c_2\sin x)(c_3\cos x + c_4\sin x)$

10. The general solution of $(x^2D^2 - xD)y = 0$ is

a) $y = c_1 + c_2e^x$ b) $y = c_1 + c_2x$ c) $y = c_1 + c_2x^2$ d) $y = c_1x + c_2x^2$

10.4.4.3 TUTORIAL TOPICS

1. Find the equation of the curve passing through the point (0, -1) and whose differential equation is $(1 + y^2)dx + (1 + x^2)dy = 0$.

2. Eliminate the arbitrary constants and obtain the DE for following.

a.$x^2 + y^2 + 2gx + 2fy + c = 0$ b.tanx tany = c (c).c.x tan(y/x) = c (c).

3. Solve $(x^2 + y^2)dx = 2xy\, dy$.

4. Solve x dy/dx + y = logx.

5. Solve dr + (2r cotθ + sin2θ)dθ=0.

6. Solve x dy/dx + y = $x^3 y^6$.

7. Solve $(1- x^2)$ dy/dx + xy = $y^3 \sin^{-1}x$.

8. Find the Orthogonal trajectories of the following family of curves.

a.$x^2/(a^2+\lambda) + y^2/(b^2+\lambda) = 1$ b.r = 2a/(1+cosθ).

9. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour what was the value of N after 1½ hours

10. A body is originally at 80°C and cools down to 60°C in 20 minutes. If the temperature of the air is 40°C, find the temperature of the body after 40 minutes

11. Solve $y^{11}+ y^1-2y = 0$ y(0) y(0) = 4, $y^1(0)$-1.

12. Solve $(D^3 +1)y = 3+5e^x$.

13. Solve $y^{11}+4y^1+4y$ = 4cosx+3sinx, y(0) = 3.

14 Solve $(x^3D^3+2x^2D^2+2)y = 10(x+1/x)$

15. $(D^2 + 3D+2)y = e^{2x}+x^2$.

16. Solve $(D^3 + 2D^2 + D)y = e^{2x}+ x^2 + x + \sin 2x$.

17. using variation of parameters solve $(D^2+1)y=\text{cosec}2x$

UNIT-V LAPLACE TRANSFORMS AND APLICATIONS TO ORDINARY DIFFERENTIAL EQUATIONS

10.4.5.1 DESCRIPTIVE QUESTIONS

1 **Find the Laplace transform of the following.**

1. cosh at (2). Sin (wt+α), α is a constant (3). $e^{2t}+4t^3-2\sin3t+3\cos3t$ (4). $\sin^2 t$
5). $\cos^3 t$ 6). t^2e^{-2t} (7). $e^{-3t}\cos4t$ (8). f (t) =t, 0<t<a =2a-t, a<t<2a
(9). $\frac{1-\cos t}{t}$ (10). $\frac{1-\cos t}{t^2}$ (11). $\frac{\sin 2t}{t}$ (12). tsinat (13). $te^{-4t}\sin3t$ (14). tsin3tcos2t
(15). $t\sin^2 3t$ (16). $e^{3t}\text{-}\frac{\sin 2t}{t}$ (17). $(e^{-at}-e^{-bt})/t$ (17). $\int_0^t e^t \frac{\sin t}{t} dt$

Find the inverse Laplace transform of the following functions

(1). $\frac{3(s-1)^2}{2s^5}$ (2). $\frac{s^2}{s^4-a^4}$ (3) $\frac{4}{(s-1)(s+2)}$ (4). $\frac{1}{s^2+25}$ 5) Log $\frac{s+1}{s-1}$
(6). $\text{Cot}^{-1}s$ (7). $\frac{s}{s^4+4a^4}$ (8). $\frac{5s+3}{(s-1)(s^2+2s+5)}$ (9). $\frac{s+2}{(s^2+4s+5)^2}$ (10) $\frac{4s+5}{(s-1)^2(s+2)}$
$\frac{s}{(s^2+a^2)^2}$ by convolution theorem (12). $\frac{s}{(s^2+a^2)(s^2+b^2)}$ by convolution theorem
(13). $\frac{1}{s^2(s^2+a^2)}$ by convolution theorem

Solve the following boundary value problems by using Laplace transforms

14. Solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$ given y=0, dy/dt =1 when t=0
15. Solve $(D^2+1)x=t\cos2t$ given x=0, $x^1(t)=0$ at t=0.
16. $y^{11}-3y^1+2y=4t+e^{3t}$ when y (0) =1 and y^1 (0) =-1
17. $ty^{11}(t)+y^1(t)+4ty(t)=0$ y(0)=3 and $y^1(0)$=-0
18. $y^{111}.+2y^{11}-y^1-2y=0$ with $y(0)=y^1(0)$=-0 and $y^{11}(0)=6$

10.4.5.2 OBJECTIVE QUESTIONS

1. The Laplace transform of $\sin^2 3t$ is
a)$3/(s^2+36)$ b)$6/(s^2+36)$ c)$18/s(s^2+36)$ d)$18/(s^2+36)$

2. $L[t^2e^t]=$
a)$2/(s-2)^2$ b)$2/(s-2)^3$ c)$1/(s-2)^3$ d)$1/(s-1)^3$ \

3. $L[e^{-t}\sinh t]=$
a) $\frac{1}{(s+1)^2+1}$ b) $\frac{1}{(s-1)^2+1}$ c) $\frac{1}{s(s+2)}$ d) $\frac{s-1}{(s-1)^2+1}$

4. Given $L^{-1}[1/(s^2+a^2)]=\frac{\sin at}{a}$,then $L^{-1}[s/(s^2+a^2)]=$
a) $\cos at$ b) $\frac{\cos at}{a}$ c) $\left(\frac{\sin at}{a}\right)^2$ d) $\frac{\sin at}{a}$

5. $L[e^{-3t}\cos 3t]=$

a) $\frac{s-3}{s^2-6s-18}$ b) $\frac{s+3}{s^2+6s+18}$ c) $\frac{s+3}{s^2-6s+18}$ d) $\frac{s-3}{s^2+6s-18}$

6. $L[(t^2+1)u(t-1)]=$

a) $2e^{-s}(1+s+s^2)/s^3$ b) $e^{-s}(1+s+s^2)/s^3$ c) $2e^{s}(1+s+s^2)/s^3$ d)None

7. $L^{-1}[e^{-s}/s^3]=$

a) $u(t-1)(t-1)^2/2$ b) $u(t-1)(t-1)^3/6$

c) $u(t).t^2/2$ d)None

8. Given $F(s)=\dfrac{1}{(s+2)(s^2+1)}$, the corresponding inverse Laplace transform, f(t) is

a) $e^{-2t}\sin t$ b) $e^{-2t}+\sin t$ c) $e^{-2t}\cos t$ d)None

9. For $L^{-1}[1/s^n]=$

a) $n>-1$ b) $n\geq 1$ c) $n=1,2,....$ d) $n<1$

10. $L\left[\int_0^t \cos u\,du\right]=$

a) $s/(s^2+1)$ b) $1/(s^2+1)$ c) $1/(s^2-1)$ d) $s/(s^2-1)$

11. $L^{-1}[1]=$

a) *t* b) *1/t* c) $\delta(t)$ d) $u(t)$

12. $L\left[\frac{\sin t}{t}\right]=$

a) $\frac{1}{s^2+1}$ b) $\cot^{-1}s$ c) $\cot^{-1}(s-1)$ d) $\tan^{-1}s$

13. The relation between unit step function and unit impulse function is

a) $L[u(t-a)]=L[\delta(t)]$ b) $L[u'(t-a)]=L[\delta(t-a)]$

c) $L[u(t)]=L[\delta'(t-a)]$ d) None

14. If $L[f(t)]=\overline{f}(s)$, then $L[f(t)u(t-a)]=$

a) $e^{-as}.\overline{f}(s)$ b) $e^{-as}.\overline{f}(s-a)$ c) $e^{-as}.L[f(t+a)]$ d) $e^{-as}.L[f(t-a)]$

15. $L^{-1}[\frac{se^{-s\pi}}{s^2+9}]=$

a) $\cos 3tu(t-\pi)$ b) $-\cos 3tu(t-\pi)$ c) $\cos 3tu(t-\pi)/3$ d)None

16. Let $L[f(t)]=\dfrac{s+1}{s^2+3}$,then *L[f(2t)]=*

a) $\dfrac{2s+1}{(2s)^2+3}$ b) $\dfrac{2(s+2)}{s^2+12}$ c) $\dfrac{s+2}{s^2+12}$ d) $\dfrac{2(s+1)}{s^2+3}$

17. $L[\sqrt{t}]=$

a) $\frac{\sqrt{\pi}}{\sqrt{s}}$ b) $\frac{1}{2}\frac{\sqrt{\pi}}{\sqrt{s}}$ c) $\frac{\sqrt{\pi}}{2s^{3/2}}$ d)Does not exist

18. A function f(t) is said to be exponential order if

a) $f(t) = e^t$ b) $f(t)e^{kt} = 1$ c) $|f(t)| \le be^{at}$ d) $|f(t)| > be^{at}$

19. The L.T. of a function *f(t)* exists of

a)It is uniformly continuous b)It is piecewise Continuous

c)It is uniformly continuous and of exponential order

d)It is piecewise continuous and of exponential order.

20. If $L[f(t)] = \overline{f}(s)$,then $L[e^{-at}\overline{f}(t)]$ is

a) $-a\overline{f}(s)$ b) $\overline{f}(s-a)$ c) $e^{-as}\overline{f}(s)$ d) $\overline{f}(s+a)$

10.4.5.3 TUTORIAL TOPICS

1. Find the Laplace transform of the following

a. $\frac{1-\cos t}{t}$ b. $e^{-3t}\cos 4t$ c. $e^{2t}+4t^3-2\sin 3t+3\cos 3t$ d. Sin (wt+α), α is a constant

2. $(e^{-at} - e^{-bt})/t$

3. $\int_0^t e^t \frac{\sin t}{t}\, dt$

4 .Find the inverse Laplace transform of the following functions

i. $\frac{3(s-1)^2}{2s^5}$ ii. $\frac{s^2}{s^4-a^4}$ iii. $\frac{4}{(s-1)(s+2)}$ iv $\frac{1}{s^2+25}$ Sin (wt+α), α is a constant

v. Log $\frac{s+1}{s-1}$ vi. $\text{Cot}^{-1}s$

5. $\frac{s}{(s^2+a^2)(s^2+b^2)}$ by convolution theorem

6. Solve the following boundary value problems by using Laplace transforms

i. Solve $(D^2+1)x = t\cos 2t$ given $x=0$, $x^1(t)=0$ at $t=0$. ii. $y^{111}.+2y^{11}-y^1-2y=0$ with $y(0)=y^1(0)=-0$ and $y^{11}(0)=6$